HILLSBOROUGH TOWNSHIP SCHOOL DISTRICT

MATHEMATICS CURRICULUM

Calculus

July, 2020

Course Overview

The Calculus curriculum is designed to provide students a thorough understanding of both Differential Calculus and Integral Calculus. Concepts will be investigated analytically, graphically, and numerically. Students will be expected to relate various representations to each other.

The course will present concepts including, but not limited to: limits, differentiation, applications of differentiation, integration and applications of integration.

Throughout the course students will use a variety of strategies and tools to gain a solid understanding of the material. Digital technology will enhance the educational experience by allowing lessons to become more interactive and inquiry-based.

The course is structured around the New Jersey Student Learning Standards. The curriculum will reflect various teaching strategies and offer opportunities for enrichment and reinforcement based on individual need.

Hillsborough Township Public Schools
Calculus CP Mathematics Curriculum

Unit Title: Unit 1 Foundations	**Timeframe/Pacing:** 15 days

Essential Questions

- What does a solution look like?
- What parts of the problem/expression are most important?
- How can I change the structure of a function or expression to make a specific form?
- What does the structure of a function tell me about its graph?

Enduring Understandings

- The structures of an equation play an important part in the process of finding a solution.
- A series of common simplifying practices leads to a common notation of a final solution.
- Each type of elementary function has a particular structure, and all transformation properties can apply to manipulate the graph.

Standards Taught and Assessed

- F-IF.B.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.C.7.d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
- A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R

Highlighted Interdisciplinary Connections

- Computer Science and Design Thinking: 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena
- ELA: SL.11-12.2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Key: ■ Major Cluster □ Supporting Cluster ◎ Additional Cluster

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity

Pre-Assessment

- Function input/output notation
- Factoring strategies/techniques
- Exponent Rules
- Label Unit Circle

Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)

- Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Find intercepts, asymptotes, domains, and graph transformations to parent functions.	SMP 7. Look for and make use of structure.	What are the asymptotes for $y = \frac{3x-2}{3x^2-4}$	Connect forms of rational relations (graph, equation, table); Revisit end behaviors by case, small group collaboration with guided examples, connect structures of equation with critical information	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Substitute quantities or numbers and simplify the difference quotient.	SMP 7. Look for and make use of structure.	If $g(x) = x^2 + \frac{1}{x+2}$, find $g(k+4)$.	Small group exercises, revisit function input, scaled examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Convert between radical and exponential forms and factor.	SMP 7. Look for and make use of structure.	Factor: $\frac{x}{\sqrt{x+2}} - \frac{1}{2}(x+2)^{3/2}$	Revisit exponent rules/definitions, scaled examples of factoring, guided small group work	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Simplify rational expressions.	SMP 7. Look for and make use of structure.	Simplify $\frac{\frac{1}{x}-\frac{1}{y}}{\frac{4}{x^2}+3}$	Small group work, revisit conditions for simplifying, compare final solutions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Model a situation and isolate a variable.	SMP 4. Model with mathematics.	Solve for n: $\frac{p}{n+1} = r$	Scaled examples, connect structures of equation to procedures of solving, teacher led examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve an equation or system of equations.	SMP 6. Attend to precision.	Solve for x exactly: $2x^3 + 4x^2 - 7x - 14 = 0$	Small group work with exercises, connect solution set to original equations	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Draw angles, find coterminal angles, and	SMP 6 Attend to precision.	Convert 140° to radians	Connect unit circle with examples, dynamic examples, teacher led	Extended time, use of calculator, challenge work and

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convert between radians and degrees.			discussion	specific other accommodations/modifications per a student's IEP or 504 plan.
Recall basic right triangle trig, exact values, period of a graph	SMP 6. Attend to precision.	If $sin\theta = -\frac{2}{3}$ and $\pi < \theta < \frac{3\pi}{2}$, then find $tan\theta$ and $sec\theta$	Small group practice with scaled exercises, compare solutions in each quadrant, connect structures of equation with critical information	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve simple trig equations over an interval	SMP 6. Attend to precision.	Find all solutions to the equation $sin(2x) = \frac{1}{2}$ on $[0, 2\pi]$.	Recall methods for solving trig equations, for common or other angles (use of inverse trig)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Not Applicable		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Not Applicable		
Summative Assessment(s) • Performance Task: Graph a rational equation, identifying all critical information, evaluating various function inputs and simplifying correctly. • Foundations Common Assessment		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

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Hillsborough Township Public Schools
Calculus CP Mathematics Curriculum

Unit Title: Unit 2 The Derivative	**Timeframe/Pacing:** 27 Days

Essential Questions

- How can you describe the behavior of a function as x approaches a certain value?
- How can we find the rate of change of a quantity? Over an interval, or at an instant?
- How can we use the derivative to solve problems?
- How can you determine the value of a limit of a function?
- How can you determine the continuity of a function at a point?

Enduring Understandings

- The derivative represents the rate of change of a quantity.
- The derivative is a function that can be obtained by various methods.

Standards Taught and Assessed

- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.
- ELA: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
- Science: HS-PS2-1: Forces and Motion Newton's second law accurately predicts changes in the motion of macroscopic objects. ("velocity").
- Computer Science and Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Key: ■ Major Cluster □ Supporting Cluster ◎ Additional Cluster

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Simplify Difference quotient • Find average rate of change of a function	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Evaluate limits or explain why the limit does not exist. (piecewise, endpoints of radicals, rational functions, factoring, one-sided limits, domain exclusions, direct substitution, conjugates)	SMP 7 Look for and make use of structure.	Given, $f(x)=\begin{cases}\frac{x^2-100}{x-10}, & x<20\\ \frac{\sqrt{x}-15}{x-225}, & 20\le x\le 400\\ \sqrt{x-800}, & x>400\end{cases}$ Find: a) $\lim_{x\to 10} f(x)$ b) $\lim_{x\to 20} f(x)$ c) $\lim_{x\to 400^+} f(x)$ If the limit does not exist, explain why.	Exploration of limits visually using Desmos or graphing calculator. Elicit analytical methods through teacher-led discussion, notes. Student practice of each method of solving.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Calculate average cost, profit, and revenue.	SMP 6. Attend to precision.	A greenhouse finds that the cost, in dollars, of growing x geraniums is	Teacher led discussion on average quantities in context.	Extended time, use of calculator, challenge work and specific other

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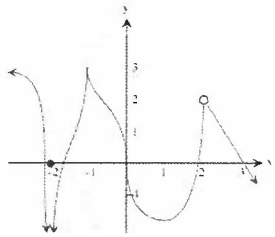
		modeled by $C(x) = 200 + 100\sqrt[4]{x}$. Find the average cost of producing the first 3000 geraniums.		accommodations/modifications per a student's IEP or 504 plan.
Determine if functions are continuous and/or differentiable.	SMP 2 Reason abstractly and quantitatively.	Identify where the graph is discontinuous and/or not differentiable. Explain why. 	Teacher led discussion of criteria for continuity and differentiability. Visualize limit values using Desmos to show instances where a function is not differentiable.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the derivative of a function using the limit definition.	SMP 7 Look for and make use of structure.	Find the derivative of $g(x) = \frac{3}{x+1}$ using the limit definition of the derivative.	Teacher-led proof of definition of derivative with examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Take the first and second derivatives (chain, quotient, product, radicals, trig)	SMP 7 Look for and make use of structure.	a) If $f(x) = 3cos^2 5x$, find f'(x). b) Find $\frac{d^2}{dx^2}(x^9 - 4x^5 + x^{-1} - 5x)$	Teacher-led examples of rules and properties of differentiation, use graphing utilities to confirm validity of rules and analytical methods, where	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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			possible (ie: expanding vs. chain rule)	
Write the equation of a line tangent to a graph at a point.	SMP 7 Look for and make use of structure.	Find an equation of the line tangent to $f(x) = \frac{5}{x^2} + \frac{x}{10} - 3$ at x=5.	Recall graphing linear equations; Apply differentiation rules to process and visualize graphically by hand and using technology	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the rate of change in a word problem.	SMP 4 Model with mathematics.	A greenhouse finds that the cost, in dollars, of growing x hundred geraniums is modeled by $C(x) = (3x^2 + 4)^5(\sqrt[4]{x} - 9x)^3$ Find the rate at which geranium cost is changing when 100 geraniums are being grown and sold.	Teacher-led discussion: Apply concept of average value functions and differentiation in a real-world context. Use of calculator to evaluate expressions.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Not applicable		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Not applicable		
Summative Assessment(s) • The Derivative Common Assessment **Performance Assessment:** Students will solve a real life application finding the rate of change of a quantity (modeled by regression capabilities)		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

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Hillsborough Township Public Schools
Calculus CP Mathematics Curriculum

Unit Title: Unit 3 Curve Sketching	**Timeframe/Pacing:** 24 Days

Essential Questions

- What do derivatives tell us about the behavior of a function?
- How can you illustrate the behavior of a function as a graph?
- How can you locate interesting or critical points on the graph of a function?
- How can you determine the curvature or concavity of the graph of a function?

Enduring Understandings

- The graph of a function can be sketched using our understanding of the derivative.
- The sketch provides useful information about the behavior of the function, including intervals of increase or decrease.
- The curve sketching methods are used to locate the minimum, maximum, and inflection points on the graph of the function.

Standards Taught and Assessed

- F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ {Performed using calculus methods}

Highlighted Interdisciplinary Connections

- Computer Science and Design Thinking: 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).
- 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity}

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Recognize graphs of familiar functions [polynomial, radical, rational, etc..]	• Extended time, use of calculator, challenge work and specific other

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		accommodations/modifications per a student's IEP or 504 plan.		
Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Find relative maximum and minimum points.	SMP 7. Look for and make use of structure.	Find all relative extrema: $f(x) = \frac{-8x}{x^2+1}$	Instruction using online textbook material, video, teacher prepared notes, and exercises. Introduce first derivative test.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find inflection points.	SMP 7. Look for and make use of structure.	Find all points of inflection: $y = x^{3/5} - 5$	Instruction using online textbook material, video, teacher prepared notes, and exercises. Introduce the second derivative test.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find vertical, horizontal, and slant asymptotes, or state that there are none.	SMP 7. Look for and make use of structure.	Find all asymptotes: a) $y = x^4 - 2x^2$ b) b) $g(x) = \frac{2x+5}{x+1}$	Recall skills from Unit 1. Incorporate into the curve sketching process.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Sketch a curve through sign analysis (chart or number line).	SMP 6. Attend to precision.	Sketch the graph of the function: $g(x) = \frac{x^2-2x+2}{x-1}$	Demonstrate the full curve sketching process; must provide exact values and use correct notation.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the absolute maximum and minimum values on a closed interval.	SMP 7. Look for and make use of structure.	Find the absolute maximum and minimum values of the function, if they exist, on the indicated interval. $f(x) = x^3 + x^2 - x + 1;\ [-2, \frac{1}{2}]$	Instruction using online textbook material, video, teacher prepared notes, and exercises. Compare methods used for open or closed intervals.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Sketch a graph given clues about the function, first derivative, and/or second derivative.	SMP 7. Look for and make use of structure.	Sketch a graph using the following information: The function is decreasing when x=-3, f"(-3)<0, f(-3)=8; f'(9)=0, the function is concave up at x=9, f(9)=-6; f"(2)=0, f(2)=1	Perform curve sketching process, given information about the function. Construct a sign chart.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply knowledge of derivatives to help solve word problems, and explain evidence of maximums or minimums using calculus vocabulary.	SMP 7. Look for and make use of structure.	The cost of widgets can be modeled by the equation $c(d) = 3d^5 - 60d + 100$, where d is the number of days that widgets have been in production and c is the cost	Primarily by example, apply methods learned to a variety of word problems.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or

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		in dollars. How long after widget production is started will the cost be a minimum? Verify that this is a minimum.		504 plan.
Describe intervals/points from a graph using calculus vocabulary.	SMP 1. Make sense of problems and persevere in solving them.	For a given graph, Identify the following intervals/points: a) Function is increasing b) Slope is increasing c) Function is positive d) $f'(x)=0$ e) $f''(x)<0$ f) $f'(x)$ undefined	Instruction using online textbook material, video, teacher prepared notes, and exercises.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Not Applicable		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Not applicable		
Summative Assessment(s) • Curve Sketching - Common Assessment • Performance Assessment: Pairs complete curve sketching, selected from several unfamiliar and more challenging functions. Present results to the class.		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

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Hillsborough Township Public Schools
Calculus CP Mathematics Curriculum

Unit Title: Unit 4 Applications of the Derivative	**Timeframe/Pacing:** 21 Days

Essential Questions

- What does an optimal solution look like?
- How can I find marginal cost, revenue, profit?
- How can I make predictions using a tangent line?
- How can I differentiate multi-variable expressions?
- How do rates of change affect one another?

Enduring Understandings

- Critical points of derivatives can locate maximum and minimum values.
- Differentials help approximate small changes of one variable based on small changes of another variable.
- Implicit differentiation is useful when finding a derivative without isolating a variable.

Standards Taught and Assessed

- F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- A-CED.A. Create equations that describe numbers or relationships

  1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

  2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

  3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods

Highlighted Interdisciplinary Connections

- ELA: RL.4.1. Refer to details and examples in a text and make relevant connections when explaining what the text says explicitly and when drawing inferences from the text
- Science: 3-5-ETS1-2 Generate and compare multiple possible solutions to a problem based on how well each is likely to meet the criteria and constraints of the problem.

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Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.2.CT.3: Use a variety of types of thinking to solve problems (e.g., inductive, deductive).
- 9.4.2.IML.2: Represent data in a visual format to tell a story about the data (e.g., 2.MD.D.10).

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations

Pre-Assessment

- Locate min/max values
- Find critical/inflection points

Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)

- Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Find $\frac{dy}{dx}$ or $\frac{dy}{dt}$ by implicit differentiation.	SMP 7. Look for and make use of structure	Use implicit differentiation to determine dx dy where $4x^3 + 2xy - y^3 = \frac{5}{2}$	Teacher led discussion, comparison of isolating a variable vs. implicit	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Approximate function values using differentials.	SMP 6 Attend to precision.	Approximate $\sqrt{63}$ to 3 decimal places	Teacher led discussion, How do calculators find square roots? Comparison of methods (calculator vs. computational)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the slope of a tangent line.	SMP 7. Look for and make use of structure	Use implicit differentiation to determine $\frac{dx}{dt}$ where	Teacher led discussion, visualizing tangent line against graph of function	Extended time, use of calculator, challenge work and specific other accommodations/modifications

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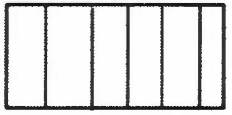
		$(2x+5)^2+y^3=\ -x,\ \frac{dy}{dt}=3$, x = -1, y = -2		per a student's IEP or 504 plan.
Optimize situations by finding maximum/minimum values through calculus.	SMP 4. Model with Mathematics	A rectangular pen is to be enclosed by a fence and divided into 6 regions as shown. Find the overall dimensions of the pen if a total of 1600 feet of fencing is to be used to obtain the largest space. 	Small group tasks, teacher led problems, connections between word problem/functions/graphs/table of values	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find marginal cost, profit, and revenue.	SMP 7. Look for and make use of structure	Let $R(x) = 50x - 0.5x^2$ and $C(x) = 4x + 10$ represent the revenue and cost in dollars, from the production and sale of x items. Find the marginal revenue, cost, and profit when x=20.	Teacher led discussion, interpret solutions to parts of the problem, connect to tangent line	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve related rates problems given a situation and values.	SMP 2 Reason abstractly and quantitatively.	The sides of a massive cube are increasing at a rate of 5 miles per year at the time when each side length is 3 miles. How fast is the volume of the cube increasing at that time?	Teacher led discussion, connect parts of problem to parts of equation, interpret and reason results	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Benchmark Assessment • Not applicable	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Not applicable
Summative Assessment(s) • Applications of Derivative Common Assessment • Performance Task: Minimize cost of manufacturing a can of soda given a fixed volume.	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Hillsborough Township Public Schools
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Unit Title: Unit 5 Exponential & Logarithmic Functions	**Timeframe/Pacing:** 22 Days

Essential Questions

- How can we find the rate of change of quantities which grow or decline exponentially?
- How does the rate of change of exponential quantities affect the actual quantity?

Enduring Understandings

- A quantity that grows exponentially grows at a rate proportional to its size.
- A logarithm can be used both to solve exponential equations as well as to find the relative rate of change of certain quantities.

Standards Taught and Assessed

- F-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior.
- F-BF.5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
- F-LE.A.4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.
- ELA: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
- Computer Science and Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

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Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Graphing exponential functions and identifying key features such as asymptotes (using limits) • Evaluate and manipulate logarithmic and exponential functions and expressions	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Find the derivative of exponential and logarithmic functions (base *e* and other bases).	SMP 7 Look for and make use of structure.	Differentiate: $g(x) = ln(x^5(4x + e^x))$ $y = x12^{2x^9}$ $y = 4e^{3x^2+5x}$	Instruction using online textbook material, video, teacher prepared notes, and exercises. Use properties of exponential and logarithmic functions to derive rules for differentiation.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve a differential equation of the form A'=k*A	SMP 2 Reason abstractly and quantitatively.	Solve: $A' = -14A$, where $A(0)=3$	Recall form of derivative of an exponential function; Compare and contrast solutions to a given differential equation.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Find the rate of change of different quantities in a real-world problem (exponential, logarithmic, limited growth).	SMP 4 Model with mathematics.	$p(t) = 1 - e^{-0.28t}$ where p is the percentage retained after t trials. Find P'(t) and explain what it means.	Teacher-led discussion of modeling and solving applications of derivatives involving exponential and logarithmic functions.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine price elasticity of demand and make conclusions about pricing.	SMP 1 Make sense of problems and persevere in solving them.	Given $q = \sqrt{200 - x^3}$, -Find the elasticity when x=3. -Will an increase in price cause total revenue to increase or decrease?	Instruction using online textbook material, video, teacher prepared notes, and exercises. Recall percent change of a quantity; relate to form of a logarithmic derivative to analyze relative rate of change of function.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Differentiation Benchmark		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • The Derivative Common Assessment **Performance Assessment:** Students will solve real life applications using derivative concepts for exponential and logarithmic functions from this unit.				

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Hillsborough Township Public Schools
Calculus CP Mathematics Curriculum

Unit Title: Unit 6 The Integral	**Timeframe/Pacing:** 17 days

Essential Questions

- What is an antiderivative?
- What meaning does an antiderivative have?
- How does a derivative relate to the original function?
- What does the area below or between two curves represent?

Enduring Understandings

- The area between the x-axis and the graph of a nonnegative, continuous function is found using a definite integral.
- The antiderivative of a function, *F,* is the series of functions that have the derivative equal to *f.*
- An initial condition is an ordered pair that is a solution of a particular antiderivative of an integrand.

Standards Taught and Assessed

- F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Highlighted Interdisciplinary Connections

- **Computer Science & Design Thinking:** 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
- **Science:** HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **ELA** SL.11-12.2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure,**

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• Points of intersection • Solving for part of equation given a solution • Finding x-intercepts		**504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Find the area between a curve and the x-axis using geometry.	SMP 4. Model with mathematics.	Evaluate $\int_0^5 4x\,dx$ geometrically.	Relate functions with geometry, teacher led discussion, small group activity	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Approximate the area between a curve and the x-axis using Riemann sums	SMP 4. Model with mathematics.	Approximate $\int_0^5 (25-x^2)dx$ using upper and lower Reimann sums with five intervals of equal length	Teacher led discussion, dynamic graphing, small group work	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the exact area between a curve and the x-axis.	SMP 4. Model with mathematics.	Find the area under $y = 3e^{6x}$ on the interval [-2,4]	Finding x-intercept;, compare geometric, reimann, integral approaches; use variety of curves	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Interpret the area between a curve and the x-axis and know concepts of integrals.	SMP 4. Model with mathematics.	If f(x) is a velocity function with units m/sec, what does the area below the curve of f(x) represent?	Connect units to functions; interpret area in terms of context; small group work; leader led discussion/exercises	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the area between two curves	SMP 6. Attend to precision.	Find the area bounded by $f(x) = x^2 - 6x + 8$ and $g(x) = 4x - x^2$	Review solving systems to find intersections; connect process to structure of definite integral; small group problems/discussions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate indefinite integrals	SMP 7. Look for and make use of structure	Evaluate $\int (3x+1)x^{-1}dx$	Connect to antiderivative, review functions that share common derivative, dynamic graphing	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate definite integrals on a closed interval.	SMP 7. Look for and make use of structure	Evaluate $\int_4^9 \left(\frac{4}{x^2} - \sqrt{x}\right) dx$	Compare/contrast to indefinite integral; relate to graphical representation; interpret meaning graphically/numerically/context of problem	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Find the antiderivative, given an initial condition.	SMP 6 Attend to precision.	Find the antiderivative f(x) given $f'(x) = x^{-1/2} + x$, and $f(4) = 10$	Review solving techniques, dynamic graphing, connect to indefinite integral	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the average value of the function over a closed interval.	SMP 7. Look for and make use of structure	Find the average value of $y = 3x^6$ over [-1,3]	Connect definition to formula/graph using multiple examples, teacher led discussion	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve problems involving marginal cost/profit/revenue or acceleration/velocity/position.	SMP 4. Model with mathematics.	A robot leaving a spacecraft has velocity $v(t) = -0.4t^2 + 2t$, where v(t) is in km/hr and t is the number of hours since the robot left the spacecraft. Find the total distance the robot traveled during the first three hours.	Connect to definite integral, interpret units and their meaning, interpret graph/area to solution.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Not applicable		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Not applicable		

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Summative Assessment(s)	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Performance Task: Given acceleration and initial conditions, find final position at specific times. • The Integral Common Assessment	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Hillsborough Township Public Schools
Calculus CP Mathematics Curriculum

Unit Title: Unit 7 Further Integration	**Timeframe/Pacing:** 19 Days

Essential Questions
- What other methods could we use to find antiderivatives?
- How can you determine what method might be used to find an antiderivative?
- Does every function that we can think of always have an antiderivative?

Enduring Understandings
- Beyond basic integration rules, there are other methods that can be used to find antiderivatives by using differentiation ideas in reverse.
- Even using a variety of methods, not all functions have an elementary antiderivative function.

Standards Taught and Assessed
- F-BF.A.1. Write a function that describes a relationship between two quantities.★ {write an antiderivative from a given function}
- A-SSE.A.1b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
- S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

Highlighted Interdisciplinary Connections
- ELA: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

Highlighted Career Ready Practices and 21st Century Themes and Skill
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Social Emotional Learning Competencies
- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).
- 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity}

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Know rules of integration for familiar functions	• Extended time, use of calculator, challenge work and specific other

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		accommodations/modifications per a student's IEP or 504 plan.		
Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Perform integration using substitution.	SMP 7. Look for and make use of structure.	$\int x^2(x^3-1)^7 dx$	Instruction using online textbook material, video, teacher prepared notes, and exercises. Recall the chain rule, and reverse..	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Perform integration by parts.	SMP 7. Look for and make use of structure	$\int xlnxdx$	Instruction using online textbook material, video, teacher prepared notes, and exercises. Recall the product rule, and reverse.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine when integration requires substitution, parts, or neither.	SMP 7. Look for and make use of structure	$\int_0^2 12xe^{0.5}dx$	Provide some general guidelines, learn by example.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Perform integration by using a reference table.	SMP 7. Look for and make use of structure	$\int \frac{1}{\sqrt{x^2-9}} dx$	Instruction using online textbook material, video,	Extended time, use of calculator, challenge work and

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			teacher prepared notes, and exercises.	specific other accommodations/modifications per a student's IEP or 504 plan.
Substitute within an integral by parts process.	SMP 7. Look for and make use of structure	$\int x\sqrt{5x+1}dx$	Provide some general guidelines, learn by example.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve word problems requiring the evaluation of a definite integral or an initial condition.	SMP 4. Model with mathematics.	Given a marginal cost function, and initial cost amount, find the total cost function.	Recall definition of marginal, learn by example.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the integral of a trigonometric function.	SMP 7. Look for and make use of structure	$\int xcosxdx$	Instruction using online textbook material, video, teacher prepared notes, and exercises.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Not applicable		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Not applicable		

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Summative Assessment(s)	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Curve Sketching - Common Assessment • Performance Assessment: Solve difficult integrals with various methods, including repeated integration by parts, or recurring integral within integration by parts	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Hillsborough Township Public Schools
Calculus CP Mathematics Curriculum

Unit Title: Unit 8 Applications of Integration	**Timeframe/Pacing:** 35 Days

Essential Questions

- What types of real world problems from various disciplines might be solved with integrals?
- How can we solve these problems if we cannot find an antiderivative for a given function?
- What would it mean for the interval of integration to be infinite? Could we solve that?

Enduring Understandings

- The integral can be used to find accumulations in various contexts, including economic and physical sciences.
- We can use numerical or algebraic approximations if we cannot find an antiderivative for a given function.
- Surprisingly, in some cases we can find the area below a curve that extends infinitely.
- We can evaluate (previously difficult) Limits and better understand the behavior of functions.

Standards Taught and Assessed

- G-GMD.A. Explain volume formulas and use them to solve problems

  1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

  2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

  3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

  B. Visualize relationships between two-dimensional and three-dimensional objects

  4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

- F-IF.C. Analyze functions using different representations

  7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Highlighted Interdisciplinary Connections

- Social Studies: 6.3.12.EconGE.1: Participate in a simulated meeting (e.g., President's Council, World Bank, International Monetary Fund, research evidence from multiple sources about an economic problem (e.g., inflation, unemployment, deficit), and develop a plan of action.

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- Science: HS-LS3-3: Algebraic thinking is used to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth).
- ELA: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).
- 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity}

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Know rules and methods of integration • Solve a compound interest problem.	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Find the equilibrium point and calculate consumer surplus and producer surplus at a point.	SMP 6. Attend to precision.	Given the demand function, $D(x) = \frac{800}{\sqrt{x+1}}$, and supply function, $S(x) = 2\sqrt{x+1}$, find: a) The equilibrium point	Instruction using online textbook material, video, teacher prepared notes, and exercises. Students taking AP Economics can provide explanations in their own words..	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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		b) The consumer surplus c) The producer surplus		
Find the accumulated future value or accumulated present value given a continuous money stream scenario.	SMP 6. Attend to precision.	Find the accumulated future value of $10 invested weekly, compounded continuously at a 0.02% weekly rate, after 5 years.	Recall compound interest. Now consider a stream of payments, as the area below the payment curve.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Calculate the total usage of natural resources and predict when said resources may run out.	SMP 6. Attend to precision.	In 2010, extraction of aluminum ore was approximately 209 million tons, and the demand was growing at a rate of 2.5% per year. The estimated world reserves in 2010 were 28 billion tons. When will the ore run out?	Natural resources may be treated similarly to financial resources.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate improper integrals or state that they are divergent.	SMP 2. Reason abstractly and quantitatively.	Evaluate $\int_2^\infty \frac{dx}{x^5}$ or state that it diverges.	Instruction using online textbook material, video, teacher prepared notes, dynamic graphing and exercises.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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Find the area under a curve, even without finite boundaries.	SMP 5. Use appropriate tools strategically.	Area under $y = \frac{4}{1+3x}$ bounded by the y-axis on the left.	Provide some general guidelines, learn by example, connect to graph	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Approximate the area under a curve using Riemann sums (left, right, or midpoint), Trapezoids, or Simpson's Rule.	SMP 5. Use appropriate tools strategically.	Approximate the value of $\int_1^4 x^{\sqrt{x}} dx$ by finding T_6	Recall definition of marginal, learn by example, relate graph/table/equation	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the volume of a solid generated by a bounded equation rotated about the x-axis. (Disc Method)	SMP 7. Look for and make use of structure.	Find the volume of the solid generated by $y = \frac{1}{\sqrt{x}}$, between x=1 and x=5 rotated about the x-axis.	Instruction using online textbook material, video, teacher prepared notes, and exercises. If possible, visit the wood shop to see a lathe in operation.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
◎ Find the volume of a solid generated by a region between two curves rotated about the x-axis. (Washer Method)	SMP 7. Look for and make use of structure.	Find the volume of the solid generated by rotating the region bounded by $y = 6 - x^2$ and $y = x^2$ about the x-axis.	Using illustration and a slight extension to the disc method.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
◎ Find the volume of a solid generated by a region	SMP 7. Look for and make use of structure.	Find the volume of the solid generated by rotating the	Use of dynamic graphing, relate common 3D objects to 2D regions of graphs,	Extended time, use of calculator, challenge work and

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between two curves rotated about any horizontal axis. (Washer Method)		region bounded by $y = 6 - x^2$ and $y = x^2$ about the line y=6.	scaled examples	specific other accommodations/modifications per a student's IEP or 504 plan.
◎ Evaluate limits using L'Hopital's Rule	SMP 7. Look for and make use of structure.	Evaluate: $\lim_{x\to\infty} \frac{x^2}{e^{3x}}$ $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$	Teacher led discussion with examples, small group work with scaled exercises. For fun, look at the Limits problem from the movie Mean Girls. Why does the limit not exist?	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
◎ Evaluate improper integrals requiring the use of L'Hopital's Rule	SMP 7. Look for and make use of structure.	Evaluate $\int_1^\infty xe^{-x}dx$	Connect methods of integration, relate structure to L'Hopital's rule, class discussion of process	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
◎ Write a Taylor Series for a given function centered at $x = a$.	SMP 8. Look for and express regularity in repeated reasoning.	Write a fourth degree Taylor Series for $f(x) = \sqrt{x}$ centered at $x = 1$.	Exploration activity in which students see the connection between building a function whose value, 1st derivative, second derivative, etc. are equal to that of a given function. Use of Desmos to compare graphs of given function and Taylor Series.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

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◎ Write a Taylor Series for a composite function using the series for an elementary Taylor Series.	SMP 8. Look for and express regularity in repeated reasoning.	Write a fourth degree Taylor Series for $f(x) = e^{x^2}$ centered at $x = 0$.	Review properties of composite functions to bridge between past knowledge and apply to new understanding. Verify reasoning graphically using Desmos or graphing utility.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
◎ Use a Taylor Series to solve a problem that cannot be solved using other techniques of integration.	SMP 1. Make sense of problems and persevere in solving them.	Look for and express regularity in repeated reasoning.Use a fourth degree Taylor Series to approximate the value of $\int_0^2 e^{x^2} dx$.	Activity to compare values obtained using this method versus those obtained using numerical integration techniques.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Cumulative Benchmark		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • Applications of Integration Common Assessment • Performance Assessment: Select one of several applications of integration..		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

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Bibliography

Supplemental Materials/Resources:

Bittinger, M. L., Ellenbogen, D. J., Scott J. Surgent, S. J., (2016). *Calculus and Its Applications Expanded Version Media Update.* Upper Saddle River, New Jersey: Pearson Prentice Hall.

Digital Resources:

www.mymathlabforschool.com